Mathematics I Midterm Exam (A)

مجموعة ( )	سكشن ( )	اسم الطالب:

(1) By Mathematical Induction prove that $\sum_{r=1}^{n}\frac{1}{3^r}=\frac{1}{2}\left(1-\frac{1}{3^n}\right)$ is true for $n\geq 1$

<u>Answer</u> The sum is true for n=1.Let the sum is true for n=k i.e. $\sum_{r=1}^{k}\frac{1}{3^r}=\frac{1}{2}\left(1-\frac{1}{3^k}\right)$

When n=k+1 $\sum_{r=1}^{k+1}\frac{1}{3^r}=\sum_{r=1}^{k}\frac{1}{3^r}+\frac{1}{3^{k+1}}=\frac{1}{2}\left(1-\frac{1}{3^k}\right)+\frac{1}{3^{k+1}}=\frac{1}{2}\left(1-\frac{1}{3^{k+1}}\right)$

The sum is true for n=k+1 then the statement is true for any positive number n.

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(2) Find the sum $\sum_{r=1}^{n}(4r^2-1)$

<u>Answer.</u> $\sum_{r=1}^{n}(4r^2-1)=\sum_{r=1}^{n}(2r-1)(2r+1)$

$$u_r=(2r-1)(2r+1)=\frac{(2r-1)(2r+1)(2r+3)}{6}-\frac{(2r-3)(2r-1)(2r+1)}{6}=f(r+1)-f(r)$$

$S_n=\frac{(2n-1)(2n+1)(2n+3)}{6}+\frac{1}{2}$ *then* $\boxed{\sum_{r=1}^{n}(4r-1)^2=\frac{(2n-1)(2n+1)(2n+3)}{6}+\frac{1}{2}}$

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(3) Find coefficient x^{20} in the expansion $(1+x+x^2+x^3+x^4)(1-x)^{-4}$.

Answer

$$(1+x+x^2+x^3+x^4)=\left(\frac{1-x^5}{1-x}\right)(1-x)^{-4}=\left(1-x^5\right)(1-x)^{-5}$$

$$=\left(1-x^5\right)\left(\sum_{r=0}^{\infty}C_r^{5+r-1}x^r\right)=\left(1-x^5\right)\left(\sum_{r=0}^{\infty}C_r^{r+4}x^r\right)\text{ Coefficient is } C_{20}^{24}-C_{15}^{19}$$

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(4) Write the first four terms in the expansion $\sqrt{4+2x}$

<u>Answer</u>

$$\sqrt{4+2x}=2(1+\frac{x}{2})^{\frac{1}{2}}=2\left[1+\frac{1}{2}(\frac{x}{2})+\frac{1}{2!}(\frac{1}{2})(\frac{1}{2}-1)(\frac{x}{2})^2+\frac{1}{3!}(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{x}{2})^3+\ldots\right]$$

$$=2\left[1+\frac{x}{4}-\frac{x^2}{32}+\frac{x^3}{128}+\ldots\right]$$